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Structural Analysis Flexible Grid Technique for SST Wing Parametric Studies

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An integrated structural analysis system has been developed for SST wing parametric studies. This integrated analysis system employs an analytical, flexible grid as the key element in combining the weights, structural flexibility, aeroelastic loads, and stress analysis into one computer program. The flexibility of the analytical grid is insured through the use of variable nodal geometrical coordinates, even though the number and relative locations of the structural nodes are fixed. A minimum of matrix input data is insured through the fixing of the number of nodes and their relative locations. This results in matrices of fixed sizes and algebraic expressions for the elements of all matrices in each of the analyses.

Nomenclature

A = diagonal matrix of structural element cross-sectional areas and thicknesses
b = transformation matrix relating final member forces to unit external loads
b_0 = transformation matrix of member forces that are statically equivalent to unit external forces
b_1 = transformation matrix of member forces that are statically equivalent to unit values of the redundant forces
F = total structural flexibility
f = structural element flexibility
R = vector (mxl) of external loads
r = external deflections caused by the external loads R
S = forces in the structural members
σ = stresses in structural elements

Introduction

THE purpose of this paper is to describe the integrated structural analysis system (Fig. 1) developed for geometry parametric studies on supersonic (SST) wing (Fig. 2). This system uses the same analytical grid for related analyses of weights, structure elastic response, aerodynamic forces, and structural member stresses.

In general, the analytical grid, which is three-dimensional, represents the following four wing elements: cover, spar, rib, and stringer. The motivation and technique whereby the mathematical definition of a grid could be made flexible enough to analyze a range of wing geometries, as shown in Fig. 2, are discussed.

Integrated Structural Analysis

Objective

One of the objectives of geometry parametric SST wing studies is the determination of the wing structural weight. Integrated structural analysis (Fig. 1) is a digital computer technique that drastically reduces the time required to determine this weight.

Problem Definition

The determination of wing structural weight proceeds from a definition of the wing external geometry and from an assumption of the required structural members and other secondary material. It was readily apparent early in SST studies that weight determination required precise analyses in the foregoing technologies. The following four primary analyses are executed to determine the structural weight: weights, structural elastic response (flexibility), aerodynamic forces, and structural member stresses. However, though the required analytical precision was available, the total time required to apply the preceding four analyses and thus obtain the structural wing weight for only one wing was excessive; for investigation of a large number of wing configurations, the time would be prohibitive. This excessive time expenditure is the result of 1) the handwritten transmission and conversion of output information from one analysis to input data for the next analysis, and 2) the time expended in each of the four

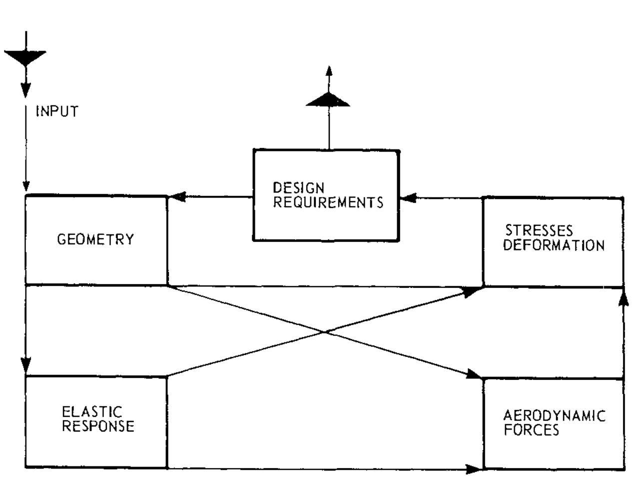

Fig. 1 Integrated structural analysis.

Received August 31, 1964; revision received March 1, 1965. In developing the programs associated with the various technologies, credit is due to E. J. Hahn (Air Loads), A. Birnbaums (Stress), and R. L. Lambert (Weights) of the SST Structures Staff.

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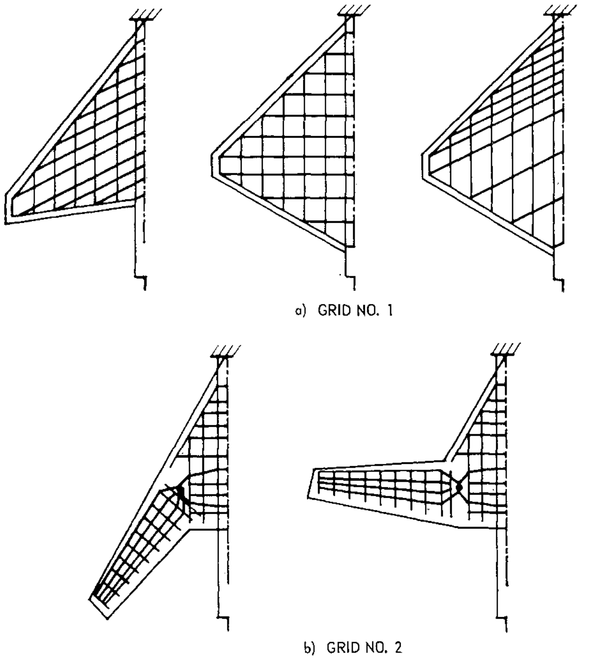

Fig. 2 Flexible structural grids.

analyses to transform initial wing geometry definitions and structural assumptions into suitable input data.

Problem Solution

The first problem to be studied was transmission of information. The four required analyses, weights, structural elastic response, aerodynamic forces, and structural member stresses, are sequential steps in the process of the determination of wing structural weight. This sequential characteristic necessitated the development of the integrated structural analysis system (Fig. 1) that defines the proper sequence of the related analyses and specifies the information flow required between analyses.

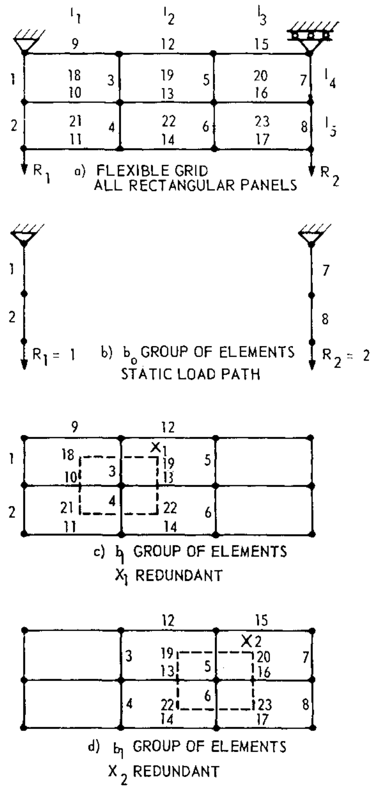

Fig. 3 Example problem, flexible grid.

The specified information flow, also shown in Fig. 1, reveals that the geometry information required by the other three major analyses in the system implies considerable economy. A single item of information could be used in three other programs. The question was: What was the factor or vehicle of commonness that could enable this implied economy to be realized?

Flexible Analytical Grids

Common grid for given wing

A review of the detail techniques being used by the previously mentioned technologies demonstrated that all resorted to a lumped parameter analysis. That is, each described a continuous phenomenon in terms of discrete phenomena at particular points over the structure. Each analysis first divided the wing into a three-dimensional grid on the basis of the same given geometry.

Thus the grid formed the basis of commonness between the various analyses in the integrated system, and the idea of the need for a single grid, common to all analyses in the system, became evident. However, a solution to the second problem, reducing the elapsed time for parametric studies, could not be effected unless some further factor of commonness between the set of wings studied could be found.

Common grid for a set of wings

A grid is characterized by the following two relationships: 1) the number of node points in the grid, and 2) the geometrical arrangement of the node points and the resulting node lines connecting particular nodes. The author's experience has shown that a fixed number of node points is sufficient for a wide range of wings. Thus there remains only the question of whether a fixed nodal arrangement could be used.

Fortunately the methods being used to determine the elastic response, stresses, and deformations would enjoy considerable computer programing advantages from a fixed grid, i.e., a grid with a fixed number and a fixed arrangement of nodal points and nodal lines. These advantages follow directly from the matrix formulations used in the structural analyses.

Flexible Grid and Structural Analysis

Basic structural analysis equations

The structural analysis that is used to determine elastic response, stresses, and deformations is the matrix force method. The fundamentals of this method and illustrations of its applicability to wings are given in Refs. 1–5.

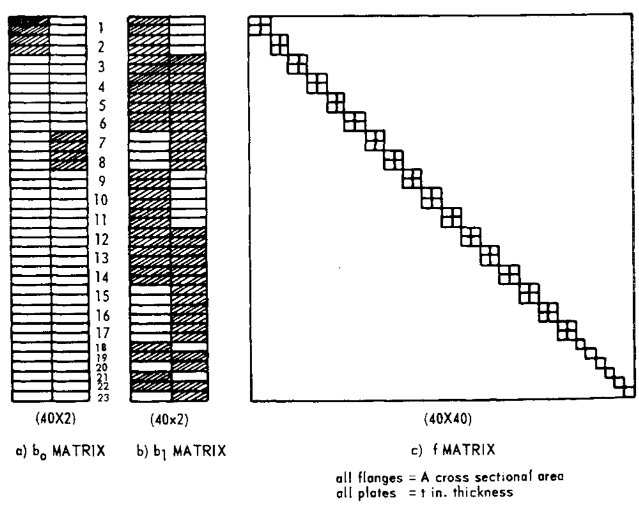

Fig. 4 Example problem, matrix patterns.

To understand the flexible grid, an examination of the pertinent matrix equations of the matrix force method is essential. For a small structure, requiring no partitioning of the matrices, the most important equations are

$$\underset{}{S} = \underset{(mxn)}{b}\ \underset{(nxl)}{R}$$

$$\underset{(mxn)}{b} = b_0 - b_1[b_1'fb_1]^{-1}\, b_1'fb_0$$

$$\underset{(nxn)}{F} = \underset{(nxm)}{b_0'}\ \underset{(mxm)}{f}\ \underset{(mxn)}{b}$$

$$\underset{(nxl)}{r} = \underset{(nxn)}{F}\ \underset{(nxl)}{R}$$

$$\sigma = \underset{(mxm)}{A^{-1}}\ \underset{(mxl)}{S}$$

It is assumed that a general purpose computer program exists to execute the solution of the preceding equations; therefore, the only remaining problem is the generation of the four matrices b_o, b_1, f, and A.

Key Factors for Computer Programing

The most important consideration in the preparation of computer programs for the generation of these matrices is the constant grid, whose configuration is still flexible. Once the number of node points is fixed, and the geometrical relationship of one node to the other is expressed in algebraic equation form, then the sizes of all matrices are known; the location, within each matrix, of the nonzero elements is explicit and unchanging, and the values of the nonzero elements of the matrices can be expressed as algebraic equations in terms of grid geometry and assumed structural material.

Example Problem

The example problem of Figs. 3–5 illustrates these three key factors. In Fig. 3a the general flexible grid and element numbering system are shown. It is seen that this flexible grid has a fixed number of nodes, 12, and the nodal configuration is specified by the lengths l_1–l_5.

For convenience only, all plates are taken as rectangles. Figures 3b–3d show the elements in the grid that are affected by the b_0 and b_1 groups (see Ref. 1 for the analysis theory). On the basis of this theory for determining the b_0, b_1, and f matrices, the patterns of the nonzero elements for these matrices can be drawn as shown in Fig. 4.

The matrix elements from b_0, b_1, and f, which correspond to structural elements 1 and 2, are shown in Fig. 5 in the algebraic form. The redundant system of Fig. 6 and the element flexibility equations of Fig. 7 are used in the b_1 and f matrices. The techniques illustrated by this example problem were readily expanded for the more complex structures shown in Fig. 2.

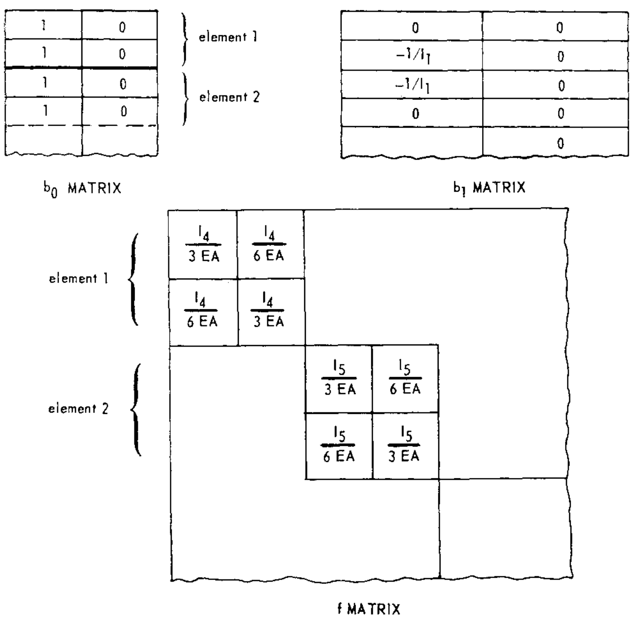

Fig. 5 Example problem, typical matrix element equations.

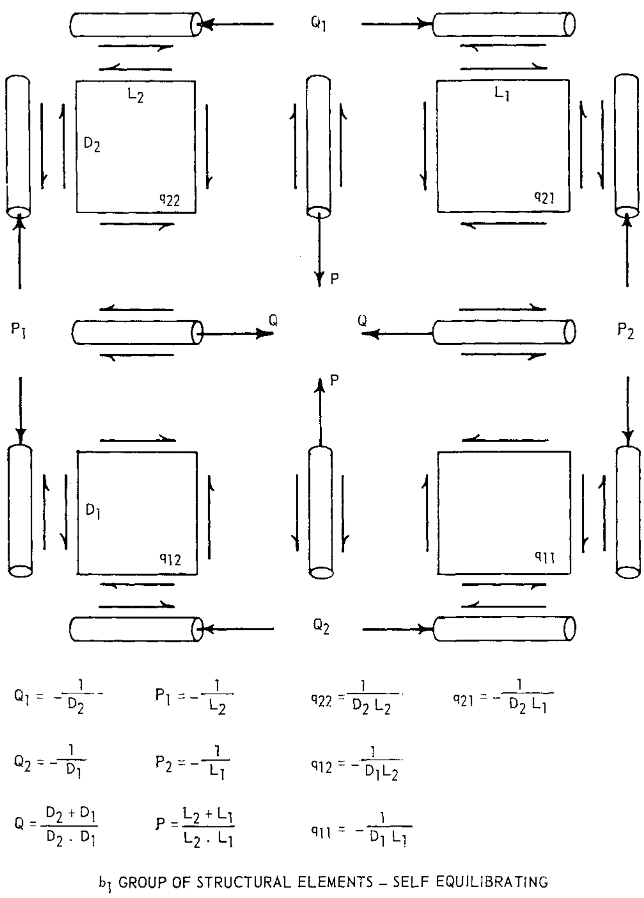

Fig. 6 Typical redundancy system.

Applications to Supersonic Aircraft

All flexible grids represent both the wing and fuselage structure as a structure system that is cantilevered from the apex of the leading edge line. Thus the fuselage flexibility and

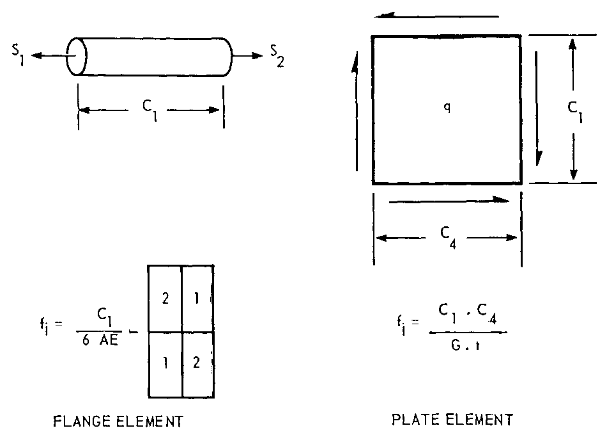

Fig. 7 Structural element flexibility matrices.

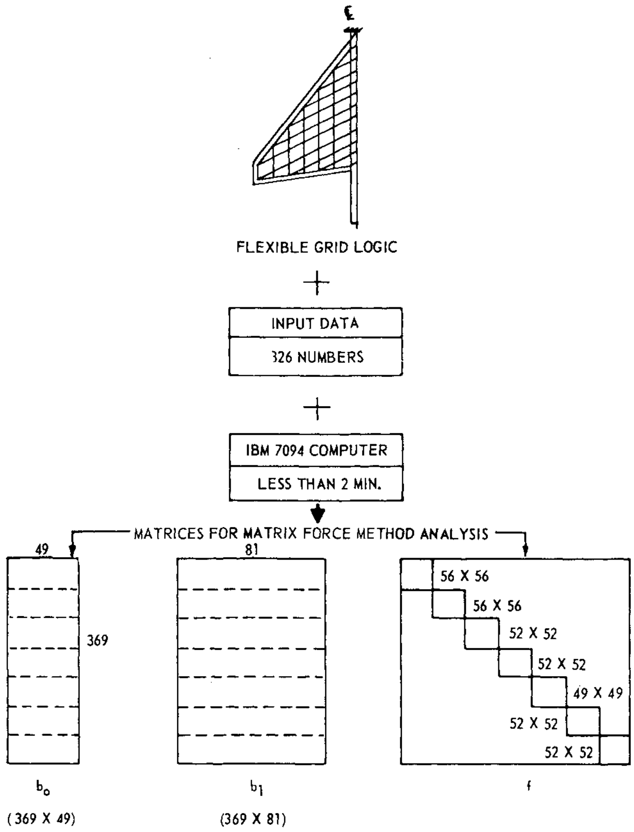

Fig. 8 Flexible grid and matrix generation.

its interaction with wing flexibility are represented by the flexible grid planforms shown in Fig. 2.

The aerodynamics, on these wing planforms, were defined for unit polynomial mode shapes both spanwise and chordwise. In the supersonic regimes a linear potential flow theory[6] was utilized, and for the subsonic regimes the work of Küchemann[7] was followed. The final airload distribution was obtained by a superposition of these unit mode shapes, and a least square fit was satisfied for the final deflected shape. Practical applications of these theories require that the center of aerodynamic pressure for the local wing panels coincide with the node points of the structural grid; and the local airloads, which are determined at selected points, should represent a good average load over this associated local aerodynamic panel.

The aerodynamic methods used associate an arbitrary shaped aerodynamic panel with each node of the flexible grid. Thus, incompatibilites between the requirements of the aerodynamic and structural grid were minimized.

Low-Aspect Ratio Wings

Grid 1 of Fig. 2 was developed for three possible variations of wing planform geometry. The ribs are equally spaced over the span dimension, and the spars are equally spaced in a forward and rearward grouping. Wing thicknesses are assumed to be symmetrical about a wing chord plane. These thicknesses can assume arbitrary values or are determined from a biconvex equation. Experience has shown that a wide range of wing geometries can be investigated with this grid.

Variable Geometry Wings

The flexible grid 2 of Fig. 2 has all the characteristics of grid 1 except that the thickness is not symmetrical about the wing chord plane. This adds considerable complexity, resulting in the wing's being represented by a set of lower surface nodes and a set of upper surface nodes. Figure 2 also shows that both the strake (inboard structure) and the outboard structure with its important trailing edge are realistically represented. The pin structure is assumed rigid in transmitting loads between the inboard and outboard structures.

Typical Input Data for Flexible Grid Programs

The generation of initial input data for the distributed weights is an integral part of integrated systems. The distributed weights are paneled to the node points of the flexible grid, by the computer program. These initial weights were computed from estimates of the primary and secondary structure. Primary structure is input as sheet gages and member areas, for spars, ribs and cover skins. Secondary structure inputs accounted for leading edge structure, trailing edge structure, and structural connections.

External and internal stores were input as distributed or concentrated weights at specified node points. The determination of wing structural weight as functions of geometry and design loading permitted considerations of airplane balance to be deferred to studies involving specific airplane configurations.

For the flexible grid 1, 326 numbers are required as weights and structures input. It is interesting to note that 11 numbers specify all wing external and internal geometry, 48 numbers are required for general material properties and generalized weight parameters covering secondary structure, engines, fuel, and miscellaneous wing contents, and 267 numbers describe the fuselage, spar, rib and cover structural materials, i.e., sheet gages, and flange areas.

The information leverage and the resulting economies in analysis elapsed time are reflected in Fig. 8. Here we see that the flexible grid logic coupled with the 326 input numbers and less than 2 min of computer time produce the three matrices necessary for the matrix force method wing analysis portions of the integrated analysis system.

Conclusions

The development of the flexible grid technique has permitted the efficient application of precise methods of structural analysis during the formative stages of SST wing design.

Economies have been effected in cost per wing analysis and in calendar elapsed time. The total input for all portions of the analysis cycle of Fig. 1 can be prepared in 8 manhours.

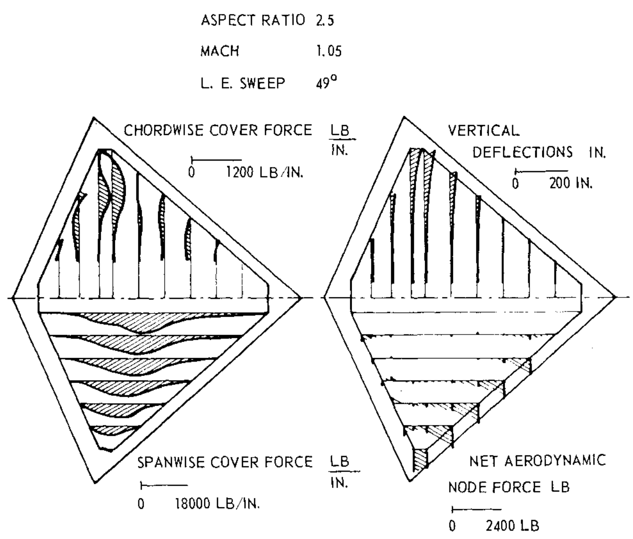

Fig. 9 Typical output from integrated analysis.

The total IBM 7094 computer time for one cycle with one design condition is 15 min. Several design conditions may be investigated during a given computer run. Each additional condition requires approximately 2 min. Our experiences indicate that two cycles of analysis are usually required to obtain a satisfactory structure. When the data for a set of wings are suitably organized, 15 wings per week can be processed.

In addition to the wing structural weight, the output from the integrated structural analysis provides a flexibility matrix for the total wing, which is the basic input for further aeroelastic, dynamic, or flutter analyses, complete distributed aerodynamic load information for subsonic and supersonic flight regimes, and detail stress information for all the structural members in the flexible grid. Figure 9 shows some plots of the typical stress and aerodynamic load output.

The flexible grid has proved to be the unifying concept whereby analyses for weights, structural elastic response, aerodynamic forces, and structural member stresses could be integrated into one computer program. The integrated analysis that is a product of the program enables rapid evaluation of wing structural weight.

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Numerical Method of Estimating and Optimizing Supersonic Aerodynamic Characteristics of Arbitrary Planform Wings

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Wing planform, which is of primary importance in its effect on drag at lifting conditions, in previous aerodynamic studies has been largely restricted to straight-line leading and trailing edges. Such restrictions, resulting mainly from a lack of analytical methods for estimating the aerodynamic characteristics of wings of arbitrary planform, may be lifted through the application of modern digital computers to the solution of the linear theory integral equations by numerical means. A numerical method for obtaining the theoretical drag-due-to-lift polars and lift-moment relationships for wings of arbitary planform and arbitrary surface shape is illustrated, and comparisons with experimental data are presented.

Nomenclature

$A(L, N)$ = leading-edge grid element weighting factor
b = wing span
C_D = drag coefficient
$C_{D,\min}$ = zero-lift drag coefficient of flat-wing configuration
ΔC_D = drag coefficient due to lift, $C_D - C_{D,\min}$
C_m = pitching-moment coefficient
$C_{m,0,F}$ = zero-lift pitching-moment coefficient of flat-wing configuration
ΔC_m = $C_m - C_{m,0,F}$
C_L = lift coefficient
$C_{L\alpha}$ = lift-curve slope per degree angle of attack
C_p = pressure coefficient
ΔC_p = lifting pressure coefficient
l = over-all length of wing measured in streamwise direction
L, N = designation of influencing grid elements
L^*, N^* = designation of field-point grid elements
M = Mach number
$\bar{R}$ = average value of influence function within a grid element
x, y, z = Cartesian coordinate system, x axis streamwise
x_{cp} = x coordinate of wing center of pressure
z_c = camber surface ordinate
Δz_c = $z_c - z_{c,le}$
α = wing angle of attack, deg
β = $(M^2 - 1)^{1/2}$
ξ, η = dummy variables of integration for x and y, respectively
τ = region of integration bounded by the wing planform and the fore Mach cone from the point x, y
Δ = wing leading-edge sweepback angle

Subscripts

F, W, WF, FF = various drag components (Fig. 7)

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Introduction

THE continuing search for aerodynamically efficient supersonic aircraft designs not only necessitates the fullest use of existing technology but also requires the development of new analytical methods of evaluating potentially efficient configurations. Because of the large portion of total